

Experimental Investigation of the
Effect of Permanent Magnetism
in Iron on the Permeability for
very small forces

Inaugural Dissertation for the degree of Doctor of
Philosophy submitted to the High Philosophical
Faculty of the University of Basel

by

DANIEL FROST COMSTOCK, S.B.

Cambridge

Printed at the University Press

1906

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Genehmigt von der mathematisch-naturwissenschaftlichen
Abteilung der philosophischen Fakultät auf Antrag der Herren
Professoren DDr. Hagenbach-Bischoff und Veillon.

PROF. DR. K. VONDERMÜHLL,
Dekan.

BASEL, *den* 11. *Juli* 1906.

I wish to take this opportunity of expressing my sincere thanks to Herr Professor Dr. H. F. Weber of Zürich for the suggestion which was the cause of this work, as well as for the advice and assistance so freely given during the progress of the same.

THE AUTHOR.

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INTRODUCTION.

THE following investigation was undertaken as the result of a suggestion of Herr Professor Dr H. F. Weber. It had as its object the determination of the effect of *permanent* magnetization on the permeability of iron for very weak forces, the fact that there existed such an effect having been previously discovered by the above-mentioned gentleman.

A minor object was to determine the relation of the magnetization to the magnetizing force for these weak forces, as there has been much dispute over this point.

It was expected that the investigation would yield some new and interesting results, and it will be seen that this expectation was justified. Beside the results pertaining to the phenomena expected, an entirely new and interesting effect was found, and it is doubtful whether it can be explained by ordinary magnetic theory.

The action of iron under weak magnetic forces presents a very interesting field for investigation, because we here deal with the iron before it has been changed to any great degree by magnetic influences.

The present work naturally divides itself into three main divisions :

I. The determination of the magnetization curves for magnetic forces between 0·01 and 0·1 and between 0·001 and 0·01.

II. The effect of permanent magnetization on the permeability for small forces.

III. The effect of *demagnetization* in steps on the permeability for small forces.

I. THE MAGNETIZATION CURVES FOR WEAK FORCES.

The subject of the effect of weak magnetic forces in producing induction has been dealt with principally by :

C. Bauer.	<i>Inaug. Diss. Zürich, 1879.</i> <i>Wied. Ann.</i> 11, 1880.
Lord Rayleigh.	<i>Phil. Mag.</i> (5), 23, 1887.
Werner Schmidt.	<i>Wied. Ann.</i> 54, 655, 1895.
G. Rössler.	<i>Inaug. Diss. Zürich, 1892.</i>
P. Culman.	<i>Wied. Ann.</i> 56, 1895.
W. Kummer.	<i>Inaug. Diss. Zürich, 1898.</i>

The first work appears to have been done by C. Bauer. He used soft iron and the ballistic method, and concluded that for small values of the magnetizing force (H) the permeability (μ) could be represented by a straight line. He therefore represented his results by an equation of the form

$$\mu = a + bH$$

where (a) and (b) are constants. From his data $a = 183$, and $b = 1382$, so that

$$\mu = 183 + 1382 H$$

and for the susceptibility (κ) we have

$$\kappa = 14\cdot5 + 110 H.$$

In his experiments (H) lay between 0·0158 and 0·384 absolute C.G.S. units.

Lord Rayleigh used the magnetometer with a compensating coil, which is then a "null method." He dealt with smaller forces, and concluded that when (H) is smaller than 0·04, the permeability is a constant, and up to 1·2 it follows an equation similar to Bauer's.

The most recent work is that of Kummer. He has measured the permeability for values of the magnetizing force

as low as 0.0001, and he finds that in no case has it a constant value. He gives for (H) between 0.001 and 0.01

$$\kappa = 8.13 + 8.3 H$$

and for (H) between 0.01 and 0.1

$$\kappa = 8.09 + 2.64 H.$$

He remarks that in the larger interval there is evidently an (H^2) term, with a small *negative* coefficient. The latter was evidently too small to measure.

THE METHOD.

Experience seems to shew that the "Ring Method" gives in general the best and most accurate results. It was used in all of the following work.

A Ring made of the iron in question has a primary coil next the core, and a series of secondary windings, any number of which can be used. A measured current is sent through the primary, and is then reversed, and the momentary induced electromotive force in the secondary is measured by means of a sensitive ballistic galvanometer. The magnetic induction of density (B) can then be calculated from the throw of the galvanometer, and the magnetizing force (H) is known from the primary current and the constants of the primary circuit. The permeability (μ) can therefore be calculated.

To shew how to determine these quantities, let us consider the circuit containing the galvanometer and the secondary coil of the ring.

Call:

R = Total resistance.

L = Coefficient of self-induction.

i = Momentary current.

E = Electromotive force induced by change in primary current.

e = Electromotive force of self-induction.

D = Galvanometer throw.

A = Area of cross-section of the ring.

N = Number of secondary windings.

The galvanometer deflection is proportional to the time integral of the current, that is

$$D = k \int i dt.$$

Now the current is of course given by

$$i = \frac{E - e}{R};$$

hence

$$\begin{aligned} D &= k \int \left\{ \frac{E - e}{R} \right\} dt \\ &= \frac{k}{R} \left\{ \int E dt - \int e dt \right\}. \end{aligned}$$

But

$$\begin{aligned} E &= \frac{d(ABN)}{dt} \\ &= AN \frac{dB}{dt} \end{aligned}$$

and

$$e = L \frac{di}{dt}.$$

$$\begin{aligned} \text{Therefore } D &= \frac{k}{R} \left\{ AN \int \frac{dB}{dt} dt - L \int \frac{di}{dt} dt \right\} \\ &= \frac{k}{R} \{ AN (B_2 - B_1) - L (i_2 - i_1) \}. \end{aligned}$$

Since the induced electromotive force is caused by a simple reversal of the primary current, it must follow that the numerical value of B_1 is equal to that of B_2 .

$$\text{Hence } B_1 = -B_2.$$

Also since the current is zero at the beginning and end of the reversal, we have

$$i_1 = i_2 = 0.$$

$$\text{Hence } D = \frac{k}{R} \{ AN 2B - L (0 - 0) \},$$

$$\text{and finally } D = \frac{2kANB}{R}.$$

To determine the constant (k) we replace the ring by an *air inductor*, the constant of which is known. We then have

$$D_1 = \frac{2kPC_1}{R},$$

where

P = constant of inductor

and

C_1 = its primary current.

(R) remains the same, since the two secondaries are left permanently in series. From the above

$$k = \frac{D_1 R}{2PC_1},$$

and hence our formula for D now stands

$$D = \frac{ANBD_1}{PC_1}.$$

Hence

$$B = \frac{PC_1 D}{AND_1},$$

which gives (B) in terms of the galvanometer deflection (D), the constant of the inductor (P), the primary current of the inductor (C_1), the total resistance of the circuit (R), the area of cross-section (A) of the magnetic circuit of the ring, the number of turns on the ring secondary, and the galvanometer deflection D_1 due to the inductor.

To calculate the magnetizing force (H), let us consider the ring.

In carrying a unit pole once around the magnetic circuit a distance of $2\pi r$ is traversed against the force (H), where (r) is the mean radius of the ring. Hence the work is $2\pi rH$.

But this must equal $\frac{4\pi}{10} NC_R$, where (C_R) is the primary current in *ampères*.

Hence

$$\frac{4\pi}{10} NC_R = 2\pi rH,$$

$$H = \frac{0.2NC_R}{r}.$$

The three necessary formulae now stand

$$B = \frac{PC_1D}{AND_1} \dots\dots\dots(\text{I}),$$

$$H = \frac{0.2NC_R}{r} \dots\dots\dots(\text{II}),$$

$$\mu = \frac{B}{H} \dots\dots\dots(\text{III}).$$

The investigation then takes the form of determining the unknown quantities in equations (I), (II) and (III).

APPARATUS.

The Apparatus used may be described as follows :

The Ring was of soft laminated Swedish iron and had the following constants :

Outer diameter	240 mm.
Inner diameter	180 mm.
Thickness	15 mm.
Number of laminations	35.
Primary windings	458.
Secondary windings	421, 100, 50, 20, 10, 5, 2, 1.
Secondary windings (wound on later)	766.

$$\text{Area} = \frac{24.0 - 18.0}{2} \cdot 1.5 = 4.5 \text{ sq. cm.}$$

The cross-section of the ring was square.

The Galvanometer was a Siemens and Halske instrument of the Thomson type, and exceedingly sensitive for this work. The changeable magnetic control made it possible to obtain whatever time of swing was desirable from $\frac{1}{3}$ second to 26 seconds. The suspension was exceedingly fine, and the torsion movement therefore very small. Since the secondary circuit apart from the galvanometer had a small resistance, the greatest sensitiveness was of course obtained with a low resistance

instrument. In the case of this instrument the resistance was about 7.5 ohms.

The mounting of the instrument was away from all mechanical disturbance, and during the entire investigation it was never touched, except to adjust the levelling screw. The control was effected by a small magnet at a distance from the instrument.

A reversing switch was used in the *galvanometer circuit* so that the deflection might always be in the same direction. This side of the scale was then calibrated by means of the inductor. The results follow:

Currents	Deflections
1786	86.63
2956	144.1
3444	167.8
3772	183.7
4208	205.5
4840	236.5

The first column is proportional to the currents, the second to the deflections after making correction for angle.

The curve is sensibly a straight line as far as the highest point. It was so considered throughout.

For primary current measurements Siemens and Halske Ampèremeters were used. In the most sensitive instrument one scale division corresponded to 0.0001 Ampère. Where it was necessary to measure even smaller currents, as was generally the case, a shunt of known resistance was used *in series* with the Ampèremeter, and the current used flowed in one arm of this shunt. By this device currents as small as 0.0001 Ampère could be measured to an accuracy of one-tenth of one per cent.

An Air Inductor was used to determine from time to time the constant of the galvanometer. It consisted of two coils, having the same axis, wound on the circumference of invariable metal rings and held apart by metal cylinders. This inductor was carefully standardized against a standard.

The secondary of the inductor was permanently joined in series with the secondary of the ring.

The current was furnished by an accumulator, and it was found that the voltage remained practically constant. A complete drawing of the apparatus and connections is shewn in Figure 1, where the following symbols are used:

- B* The Ring.
- A* The Galvanometer.
- N* The Ampèremeter.
- M* The Accumulator.
- I* The Air Inductor, the primary and secondary of which are represented by (*W*) and (*X*) respectively.
- R₁R₂* The resistance box used to reduce the current used.
- T* The switch in the primary used to reverse the current at each measurement.
- L* Switch used to send the current through primary of ring or inductor at will.
- V* Resistance in primary circuit to regulate primary current.
- K* Resistance in galvanometer circuit used to regulate the sensitiveness of the galvanometer.
- D* Switch used in order that the galvanometer deflection should always be in the same direction.

METHOD OF MAKING OBSERVATIONS.

The observations were made in the following way:

At the beginning, at the end, and as many times as necessary during each run, the galvanometer constant was determined. To do this a steady current was sent through the primary of the inductor and the throw of the galvanometer was noted when this current was reversed. The current, the galvanometer throw and the resistances *R₁* and *R₂* were then recorded. This was repeated from 4 to 12 times; from these data the galvanometer constant can be found.

Similar observations were then made when the current was sent through the *ring* primary. The necessary values for the current corresponding to the desired values of the magnetizing force were calculated beforehand.

The time of each set of observations was also recorded, in order that any gradual change in the galvanometer constant could be allowed for.

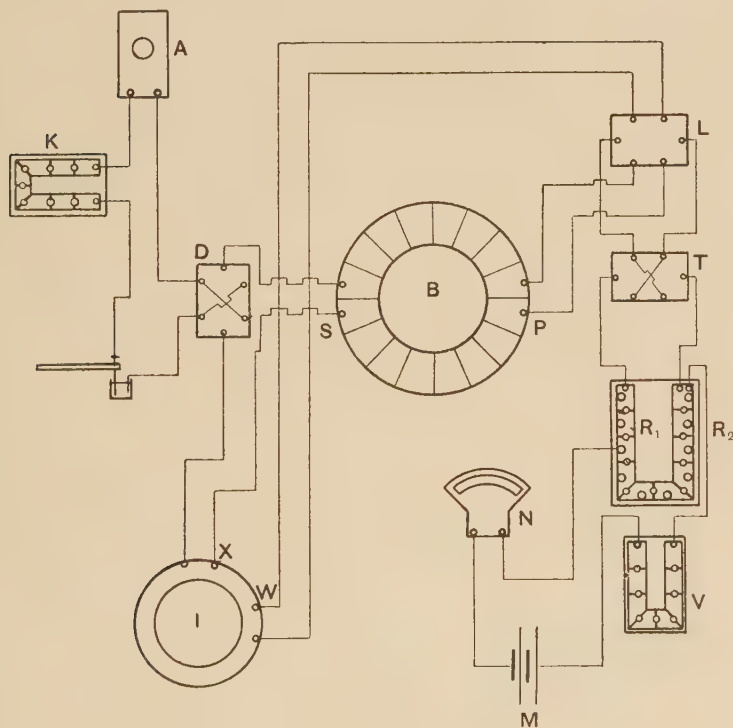


Fig. 1.

PRECAUTIONS AND DETAILS.

Great care was taken in all the observations that no error or disturbing cause should come into play. Before each run the galvanometer was examined to see if it were accurately level, and the instrument was carefully protected from all outside influences. The galvanometer was so sensitive that it was

found that when the control was weakened until the time of a half swing was much greater than 5 sec., the zero unsteadiness was too great. Five seconds was therefore used as time of swing in practically all of the work.

It was, of course, necessary to avoid the effects of thermal currents. The switch (*D*) in the secondary was therefore enclosed in a pasteboard box, and the terminals wrapped with cotton.

It was found that the action of the inductor had an effect on the galvanometer, and therefore it was placed in another room where the distance was much greater.

Since the resistance R_2 is relatively small, an error in the current measurement would be made if the connections were not particularly good. This was always carefully considered.

A Correction for change of resistance of the secondary circuit comes into play when the number of secondary windings is changed. As seen from the formula on page 4, namely

$$D = 2k \frac{NAB}{R},$$

the galvanometer deflection varies inversely as the resistance of the secondary circuit. A simple correction for the change in resistance due to the change in the windings could therefore be made, if it were not for the fact that the *damping* of the galvanometer is also changed by the change in resistance. This effect is small, but because of it, and because of the inconvenience of measuring the value of so small a resistance to the necessary degree of accuracy, the following plan was adopted. During any run the same number of secondary windings were always used in series, and when a smaller deflection was desired, it was obtained by changing the terminals of some of the coils so that their effect was subtractive, although the resistance remained the same. In this way all such corrections were avoided.

The value of the magnetizing force is not of course absolutely the same at all distances from the axis of the ring, but the value obtained when we use for the radius the

arithmetic mean of the least and greatest radii may be considered a good representative value.

A change of a few degrees in the temperature of the room would evidently change the galvanometer constant for

$$D = \text{constant} \cdot \frac{1}{R},$$

and since for pure metals the resistance varies 0.004 per degree we have

$$\frac{\delta D}{D} = 0.004 \delta T,$$

where (δT) is the change in Temperature. A change of 3° in temperature would therefore affect the galvanometer constant by more than one per cent., and under ordinary circumstances such a change could easily take place during a run of several hours.

In the present work, however, unusually good temperature conditions were obtained, for the room was below ground level, and experiment shewed that the temperature scarcely varied one-half of a degree during the entire afternoon.

Through most of the work care was taken always to change the magnetic forces in the same direction, so that no lag in the magnetization could cause error.

The air inductor used was standardized against a standard inductor by taking a series of galvanometer readings when the secondary coils of the two were first together and then in opposition, while the same current went through the primaries. Calling (a) and (b) the readings when they were together and in opposition respectively, and P and P_0 the constants of the coils, we have

$$\frac{P}{P_0} = \frac{a-b}{a+b},$$

or
$$P = P_0 \frac{a-b}{a+b}.$$

The mean of two series of sixteen readings in each series gave

$$a = 118.03,$$

$$b = 45.985,$$

and the constant of the standard was given as

$$P_0 = 0.3974 \cdot 10^3.$$

Hence

$$\mathbf{P = 0.17457 \cdot 10^3},$$

and this was the value used.

CALCULATION AND RESULTS.

As has been stated on page 6 the results were calculated from the formulae

$$B = \frac{PC_1}{AN_s D_1} \cdot D,$$

$$H = \frac{0.2 N_P C_R}{r},$$

$$\mu = \frac{B}{H}.$$

Now

$$\begin{aligned} P &= \text{Air inductor constant} \\ &= 0.1746 \cdot 10^3. \quad (\text{See above.}) \end{aligned}$$

$$\begin{aligned} A &= \text{Area of Ring cross-section} \\ &= 4.5 \text{ cm.}^2 \quad (\text{See page 6.}) \end{aligned}$$

$$\begin{aligned} N_P &= \text{Number of primary windings} \\ &= 458. \quad (\text{See page 6.}) \end{aligned}$$

$$\begin{aligned} r &= \text{Mean Radius of Ring} \\ &= 10.5 \text{ cm.} \quad (\text{See page 6.}) \end{aligned}$$

Also

$$N_s = \text{Number of secondary windings used.}$$

$$C_R = \text{Current through ring primary in Ampères.}$$

$$C_1 = \text{Current through inductor primary in Ampères.}$$

$$D_1 = \text{Galvanometer deflection caused by inductor.}$$

$$D = \text{Deflection caused by ring.}$$

The following (Table I) is given to represent such a calculation. The simple corrections such as that for reducing deflections to angular values are not given, but everything else is.

TABLE I.

Ampèremeter $\times 10^4$	Ratio	H	D	D ₁	N _s	B	μ
101.7*	200/1016			217.9**			
145.6	200/1000	0.02540	112.9	217.8	85	4.739	186.6
148.4	300/1000	0.03885	182.6	217.8	85	7.663	197.2
124.2	500/1000	0.05419	175.5	217.8	55	11.380	210.0
81.4	1	0.07101	154.6	217.8	35	15.750	221.7
97.9	1	0.08541	194.4	217.8	35	19.820	232.0
116.8	1	0.10190	174.0	217.8	25	24.820	243.6
134.8	1	0.11760	208.4	217.8	25	29.740	252.8
101.8*	200/1016			217.9**			

Note:—In the tables which follow, the numbers are in general carried to four figures, and zeros added for symmetry.

It corresponds to forces between $H = 0.01$ and $H = 0.1$.

The numbers marked (*) are ampèremeter readings for (C_1), and were used in finding the galvanometer constant. If the upper and lower values of C_1 differed, the upper one was always used, and the deflections reduced. The numbers marked (**) are the deflections corresponding to the readings marked (*).

The results of the best series of measurements for values of H between 0.01 and 0.1 are given below, as are also the values of (μ) calculated from the derived equation.

The susceptibility is called (x).

TABLE II.

H	B	μ	x	Cal. μ	$\delta\mu$
0.02540	4.739	186.6	14.74	187.3	+ 0.7
0.03885	7.663	197.2	15.61	197.5	+ 0.3
0.05419	11.380	210.0	16.63	209.8	- 0.2
0.07101	15.750	221.7	17.56	222.5	+ 0.8
0.08541	19.820	232.0	18.38	233.2	+ 1.2
0.10190	24.820	243.6	19.31	242.9	- 0.7
0.11760	29.740	252.8	20.04	253.1	+ 0.3

Average deviation of calculated from observed values = 0.3%.

EQUATIONS WHEN H LIES BETWEEN 0.01 AND 0.1.

$$\mu = 167.0 + 816H - 700H^2.$$

$$B = 167.0H + 816H^2 - 700H^3.$$

$$x = 13.20 + 64.9H - 55.7H^2.$$

$$I = 13.20H + 64.9H^2 - 55.7H^3.$$

METHOD OF OBTAINING THE EQUATIONS.

The method of least squares furnishes a means of determining the equation of a curve when the observed data are given. The Method of Gauss could also be used to advantage. The equations were, however, obtained by an entirely different method, which furnishes equally good results.

TABLE III.

Ampèremeter $\times 10^4$	Ratio	H	D	D ₁	N _S	B	μ
103.3*	200/1016			1938**			
107.5	10/1000	0.000938	506.8	1937	1287	0.1605	171.0
141.3	20/1000	0.002465	1331.0	1937	1287	0.4211	170.8
115.5	40/990	0.004071	2210.0	1937	1287	0.6992	171.7
125.2	50/1000	0.005462	1029.0	1937	445	0.9416	172.4
120.1	60/960	0.006548	1239.0	1937	445	1.1340	173.2
135.8	70/970	0.008549	1629.0	1937	445	1.4900	174.4
145.5	80/980	0.010360	1985.0	1937	445	1.8170	175.4
140.4	100/1000	0.012250	1300.0	1937	245	2.1600	176.4
103.3*	200/1016			1938**			

The permeability was plotted and the point where the curve produced cuts the axis of (μ) gives us the value for (a) in the equation

$$\mu = a + bH + cH^2.$$

The value for $\frac{(\mu - a)}{H}$ was then found for a number of points, and was then plotted with (H) . This gave a *straight line*, as is evident it should from the equation

$$\frac{\mu - a}{H} = b + cH.$$

This straight line cut the axis at the distance 816, so that

$$b = 816.$$

The line sloped downward, and to the right, and had a slope corresponding to

$$c = -700.$$

In this way the three constants were determined and the method is shewn to be a good one by the small deviation from the observed results of only 0.3%.

RESULTS FOR H BETWEEN 0.001 AND 0.01.

Above (Table III) are given the calculations for the values of H between the above limits, the computation being carried on as described on pages 12 and 13.

Table IV gives the results and the deviation of the calculated from the observed values. In the case of these very

TABLE IV.

H	B	μ	x	Cal. μ	$\delta\mu$
0.000938	0.1605	171.0	13.52	—	—
0.002465	0.4211	170.8	13.51	170.8	0.0
0.004071	0.6992	171.7	13.58	171.8	+ 0.1
0.005462	0.9416	172.4	13.63	172.7	+ 0.3
0.006548	1.1340	173.2	13.70	173.2	0.0
0.008549	1.4900	174.4	13.79	174.4	0.0
0.010360	1.8170	175.4	13.88	175.4	0.0
0.012250	2.1600	176.4	13.96	176.5	+ 0.1

small forces, an equation of the first degree served to represent accurately the permeability (μ).

EQUATIONS ($0.002 < H < 0.01$).

$$\mu = 169.4 + 580H.$$

$$B = 169.4H + 580H^2.$$

$$x = 13.40 + 46.1H.$$

$$I = 13.40H + 46.1H^2.$$

Average deviation of calculated values from observed values = 0.05 %.

DISCUSSION OF RESULTS.

From the foregoing it is evident that for forces between 0.02 and 0.1 the permeability is *not* a constant as Rayleigh supposed, nor is it to be expressed as a linear function as others have thought. It can however be expressed quite accurately as a *quadratic function*, and the flux density (B) therefore as a *cubic function* of (H).

Kummer noticed that the function was probably of this form, and that the last constant had "a small negative value" as he said, but he did not attempt to give a value to this constant, probably because it was so hard to determine.

For forces between 0.002 and 0.01, the permeability seems to be remarkably well represented by a *linear* function, the deviation being only 0.05 %, but this line is *not* a horizontal one.

Apparently then the permeability is *never a constant* even when the forces are as small as those here dealt with.

At the lowest value for (H) in most of the runs, the permeability seemed to be higher than the calculated value. This may have been due to the fact that this observation was always difficult because of the exceedingly small effect.

POINT OF INFLECTION OF THE CURVE.

An inspection of the two equations,

$$\mu = 167.0 + 816H - 700H^2 \quad \text{for } 0.1 > H > 0.02$$

and $\mu = 169.4 + 580H \quad \text{for } 0.01 > H > 0.002,$

shews that the first curve *rises more rapidly* than the second.

(H^2) has however a *negative* coefficient, and therefore tends to rise less rapidly as (H) increases. Therefore it follows that there must be a point of inflection in the region near the junction of the two curves.

Thus the curve joining the permeability with the force (H) is by no means as simple as the early investigators would like to prove.

II. THE EFFECT OF PERMANENT MAGNETIZATION ON THE PERMEABILITY.

The investigation of the subject now to be considered was in reality the principal purpose of the entire work.

Soft iron in the ring form retains much of the magnetism given to it, and it was desired to test the effect of it on the permeability for small forces.

The apparatus used to test this was that described in Section I.

DEMAGNETIZATION OF THE RING.

The investigation of the effect mentioned naturally begins with the complete demagnetization of the ring, and this was accomplished by passing through the primary of the ring a fairly strong alternating current which was reduced in steps until the ring was completely demagnetized. About ten minutes was allowed between each reduction.

This process was found very effective, and was therefore used throughout when it was desired to demagnetize the ring.

RESULTS ON THE DEMAGNETIZED RING.

The permeability of the ring in this condition was measured as was described in the former cases, and the results obtained are tabulated below (Table V) for forces between 0.001 and 0.01, and in Table VI for forces between 0.01 and 0.1.

The equations representing these results were obtained as explained on page 14, and are here given. As before, the

first observation in the series of $H = 0.001$ to 0.01 is omitted in the consideration.

$$\mu = 188.9 + 360H.$$

$$B = 188.9H + 360H^2.$$

$$x = 14.95 + 28.6H.$$

$$I = 14.95H + 28.6H^2.$$

Average deviation of calculated from observed values = 0.08% .

TABLE V.

H	B	μ	Cal. μ	$\delta\mu$	x
0.000923	0.1762	190.7	189.2	—	15.10
0.002482	0.4718	189.8	189.8	0.0	15.03
0.004077	0.7773	190.4	190.4	0.0	15.07
0.005457	1.0410	190.6	190.9	+ 0.3	15.09
0.006564	1.2560	191.5	191.3	— 0.2	15.16
0.008572	1.6480	192.4	192.0	— 0.4	15.23
0.010370	1.9960	192.6	192.6	— 0.0	15.24
0.012110	2.3450	193.7	193.3	— 0.4	15.33
0.013010	2.5190	193.6	193.6	— 0.0	15.32

TABLE VI.

H	B	μ	x
0.02522	5.141	203.8	1614
0.03889	8.362	215.0	1702
0.05575	12.770	229.0	1813
0.07015	16.840	240.1	1902
0.08610	21.580	250.7	1984
0.10200	26.730	262.1	2077
0.10960	29.390	268.1	2125

The equations for Table VI are

$$\mu = 184.1 + 814H - 480H^2.$$

$$B = 184.1H + 814H^2 - 480H^3.$$

$$x = 14.56 + 64.8H - 38H^2.$$

$$I = 14.56H + 64.8H^2 - 38H^3.$$

It is evident from the results that the permeability has *risen* considerably because of the demagnetization.

The former equation when H is between 0.001 and 0.01 was

$$\mu = 169.4 + 580H,$$

the present one corresponding to demagnetization

$$\mu = 188.9 + 360H.$$

The constant (a) has therefore risen more than 10%.

A similar result follows from an inspection of the curves for forces from 0.01 to 0.1.

$$\text{Formerly} \quad \mu = 167.0 + 816H - 700H^2.$$

$$\text{Now} \quad \mu = 184.1 + 814H - 480H^2.$$

In this case the constant (a) has risen also about 10%.

In the latter equations the constant (b) has remained about the same.

That is

$$\text{Formerly } a = 167.0, \quad \text{Now (demagnetized) } a = 184.1,$$

$$b = 816, \quad b = 814,$$

$$c = -700, \quad c = -480.$$

MAGNETIZATION WITH FORCE OF 16.7 C.G.S. UNITS FOR ONE MINUTE.

After the measurements on the demagnetized ring a current of 1.918 ampères was allowed to flow through the primary for one minute. This corresponds to a magnetizing force of 16.7 absolute C.G.S. units. The force was then withdrawn and the permeability was again measured as has been explained, and the following results obtained.

EQUATIONS.

$$\mu = 107.8 + 234H.$$

$$B = 107.8H + 234H^2.$$

Average deviation of calculated from observed values = 0.12 %.

TABLE VII.

(*Magnetizing force used* = 16.7.)

<i>H</i>	<i>B</i>	μ	Cal. μ	$\delta\mu$
0.0009152	0.09902	108.1	108.0	-0.1
0.0024860	0.26980	108.5	108.4	-0.1
0.0040360	0.43900	108.7	108.7	0.0
0.0054880	0.59780	108.9	109.1	+0.2
0.0065430	0.71420	109.1	109.3*	+0.2
0.0085430	0.93760	109.7	109.8	+0.1
0.0104000	1.14800	110.4	110.2	-0.2
0.0122200	1.35500	110.9	110.7	-0.2
0.0129500	1.43400	110.9	110.8	-0.1

It is evident that the permeability has dropped about 43 % because of the magnetization. It is still represented by a straight line, but the slope (b) = 234 is not apparently as great as before.

It was found impracticable to measure the permeability for the strongly magnetized ring for forces between 0.01 and 0.1 because the reversals of the measuring forces disturbed the permanent magnetism. The smaller forces were therefore exclusively used in the following.

MAGNETIZATION WITH FORCE OF 25.9 C.G.S. UNITS
FOR 55 SECONDS.

The ring was next magnetized by the use of 2.976 ampères (corresponding to a force of 25.9) for 55 seconds.

The permeability was then once more measured with the following results.

EQUATIONS.

$$\mu = 108.5 + 183H.$$

$$B = 108.5H + 183H^2.$$

Average deviation of calculated from observed values = 0.2 %.

TABLE VIII.

(*Magnetizing force used* = 25.9.)

<i>H</i>	<i>B</i>	μ	Cal. μ	$\delta\mu$
0.0009080	0.09865	108.6	108.7	+ 0.1
0.0024970	0.27200	109.0	109.0	0.0
0.0040470	0.44240	109.6	109.2	- 0.4
0.0054790	0.59940	109.4	109.5	+ 0.1
0.0065100	0.71200	109.4	109.7	+ 0.3
0.0084920	0.93350	109.9	110.1	+ 0.2
0.0104100	1.15100	110.6	110.4	- 0.2
0.0122600	1.36000	111.0	110.7	- 0.3
0.0131100	1.45300	110.8	110.9	+ 0.1

It is evident that although there was at first a very great drop in the permeability, there is now practically no change.

This led to the belief that the curve was asymptotic, and that the limiting value of the permeability had been practically reached.

To test this the following measurements were made.

MAGNETIZATION WITH FORCE OF 52.9 C.G.S. UNITS
FOR ONE MINUTE.

A current of 6.06 ampères (corresponding to a force of 52.9) was allowed to flow for one minute, and the permeability measured with the following results.

TABLE IX.

(Magnetizing force used = 52.9.)

<i>H</i>	<i>B</i>	μ	Cal. μ	$\delta\mu$
0.002484	0.2726	109.7	109.8	+ 0.1
0.004058	0.4465	110.0	110.1	+ 0.1
0.005488	0.6054	110.4	110.3	- 0.1
0.006569	0.7269	110.6	110.4	- 0.2
0.008523	0.9382	110.0	110.6	+ 0.6
0.010400	1.1510	110.8	110.9	+ 0.1
0.012540	1.3950	111.3	111.1	- 0.2
0.013100	1.4560	111.1	111.2	+ 0.1

EQUATIONS.

$$\mu = 109.6 + 122H.$$

$$B = 109.6H + 122H^2.$$

Average deviation of calculated from observed values = 0.2 %.

The last magnetizing force used was double the previous one and three times the one before that, but in spite of this fact the permeability has remained practically constant. It is interesting to note, however, that the constant (*b*) is apparently becoming less.

MAGNETIZATION WITH FORCE OF 2.79 C.G.S. UNITS
FOR ONE MINUTE.

In order to obtain information with regard to the action for smaller magnetizations the ring was once more demagnetized by the use of an alternating current as before mentioned, and then magnetized with a current of 0.320 ampère (corresponding to a force of 2.79) for one minute.

The results were as follows.

TABLE X.

(Magnetizing force used = 2.79.)

<i>H</i>	<i>B</i>	μ	Cal. μ	$\delta\mu$
0.0009272	0.1259	135.7	135.6	- 0.1
0.0024740	0.3364	136.0	136.0	0.0
0.0040540	0.5520	136.1	136.4	+ 0.3
0.0054960	0.7509	136.7	136.8	+ 0.1
0.0065750	0.9012	137.1	137.1	0.0
0.0085860	1.1820	137.7	137.6	- 0.1
0.0104500	1.4440	138.1	138.1	0.0
0.0125200	1.7420	139.0	138.7	- 0.3

EQUATIONS.

$$\mu = 135.5 + 258H.$$

$$B = 135.5H + 258H^2.$$

Average deviation of calculated from observed values = 0.1 %.

Here it is evident that the permeability has risen about 25 %.

The constant (*b*) is also larger, that is to say, the line has a greater slope.

MAGNETIZATION WITH A FORCE OF 5.95 C.G.S. UNITS
FOR ONE MINUTE.

Another run was made by using a current of 0.682 ampère, which corresponds to a force of 5.95.

The results were as follows.

EQUATIONS.

$$\mu = 116.7 + 180H.$$

$$B = 116.7H + 180H^2.$$

Average deviation of calculated from observed values = 0.15 %.

TABLE XI.

(Magnetizing force used = 5.95.)

H	B	μ	Cal. μ	$\delta\mu$
0.0009247	0.1081	116.9	116.9	0.0
0.0024740	0.2895	117.0	117.1	+ 0.1
0.0040680	0.4785	117.6	117.4	- 0.2
0.0054740	0.6431	117.5	117.7	+ 0.2
0.0065980	0.7767	117.7	117.9	+ 0.2
0.0085490	1.0130	118.5	118.2	- 0.3
0.0104400	1.2390	118.6	118.6	0.0
0.0125200	1.4890	118.9	119.0	+ 0.1
0.0132400	1.5760	119.0	119.1	+ 0.1

MAGNETIZATION WITH FORCE OF 10.9 C.G.S. UNITS
FOR ONE MINUTE.

One more run was made with a current of 1.247 ampères, which corresponds to a force of 10.9.

TABLE XII.

(Magnetizing force used = 10.9.)

H	B	μ	Cal. μ	$\delta\mu$
0.002493	0.2728	109.4	109.3	- 0.1
0.004086	0.4466	109.3	109.7	+ 0.4
0.005457	0.6003	110.0	109.9	- 0.1
0.006580	0.7244	110.1	110.2	+ 0.1
0.008572	0.9515	111.0	110.6	- 0.4
0.010380	1.1520	111.0	111.0	0.0
0.012170	1.3490	110.9	111.4	+ 0.5
0.012990	1.4430	111.1	111.5	+ 0.4

EQUATIONS.

$$\mu = 108.8 + 210H.$$

$$B = 108.8H + 210H^2.$$

Average deviation of calculated from observed results = 0.25 %.

EFFECTS OF MECHANICAL VIBRATION.

It was found that the slightest jar on the ring when highly magnetized produced a throw in the galvanometer. A simple finger tap caused a throw of a number of millimetres, and a sharp rap with a small block of wood would send the zero nearly off the scale.

To avoid trouble from this source, while the experiments were being made the ring was mounted on a heavy roll of cloth, and hence put out of contact with any solid substance which could communicate a jar to it. During the latter part of the experiments the ring was never touched during readings.

DOWNWARD DRIFT OF THE READINGS.

At practically all times when the ring was magnetized, and especially when this magnetization was high, a decided downward drift of the individual readings was apparent. The readings began *after* the current had already been reversed about thirty times. The effect cannot therefore be due to any cause at work when the first few reversals were made. This was no doubtful phenomenon, but seemed to occur definitely and almost continually. The readings would decrease more and more slowly, and often there was a definite limit which they seemed to approach.

Below, in Table XIII, is given an example taken from the data. It represents point five in Table IX, that is to say, when the ring was magnetized with a force of more than 50 C.G.S. units, and when the small force used in measuring was about 0.085. The two columns correspond to throws of the reversing switch from left to right and from right to left respectively. The zero of the scale was at 500.0.

This is not an isolated case, but shews a general tendency when the ring was strongly magnetized.

It will be seen later that this effect, slight though it is, is by no means easy to explain, because most of the effects naturally thought of would cause the readings to *increase*, instead of *decrease* as they do.

TABLE XIII.

709·3	710·0
708·8	709·7
708·3	709·1
708·2	708·2
708·2	707·8
707·8	708·1
707·8	708·0
707·7	707·5
707·8	707·7
707·3	707·7
707·7	707·2
707·7	707·8

In Table XIV is given a further example of the same phenomenon. *Before these readings began*, the current was reversed

20 times at 1 second intervals

then 6 „ „ 10 „ „
 then 4 „ „ 20 „ „

This series, like the other, is only an example. Many others might be given.

It will be observed that the column to the right in both tables decreases faster than the other. It reaches the same value in the end, having started with a larger one.

TABLE XIV.

670·4	671·8
670·7	670·5
669·6	670·9
669·9	670·7
669·6	669·8
669·8	670·0
669·4	670·4
669·8	670·1
670·0	670·1
670·0	670·1
669·3	670·1
670·1	—

DISCUSSION OF RESULTS.

A glance at the results furnished in the preceding section reveals a number of interesting facts.

It is evident that in general the effect of permanent magnetism is to lower the permeability for small forces. There is a limiting value, however, below which the permeability cannot go, even if the magnetizing force is greatly increased.

When the iron is magnetized, the *form of the function* connecting the induction and the small forces used does not change greatly. It does, however, to a small degree.

Below, in Table XV, are given the values of the large force used to magnetize, with the corresponding values of (*a*) in the equation

$$\mu = a + bH.$$

TABLE XV.

Magnetizing Force	Value of (<i>a</i>)	Reference
0.00	188.9	Table V
2.79	135.5	Table X
5.95	116.7	Table XI
10.90	108.8	Table XII
16.70	107.8	Table VII
25.90	108.5	Table VIII
52.90	109.6	Table IX

From the above it is seen that as the magnetizing force is increased, the permeability decreases rapidly at first, and then slowly reaches a minimum value. It then seems to increase very slowly towards the last.

THE VARIATION IN THE *FORM* OF THE
PERMEABILITY FUNCTION.

An inspection of the results obtained reveals the fact that, although the permeability function remains linear, yet the slope of the line changes when the iron is magnetized. This variation

is very difficult to determine because the quantity itself is so small. Great accuracy can therefore not be expected, but there is a general tendency to *decrease* with increasing magnetization.

Below, in Table XVI, are given the magnetizing forces, and the corresponding values of (b) in the equation

$$\mu = a + bH.$$

An error of 0.3% in the permeability corresponds to a change of about 15% in the constant (b) , and therefore the concordance of the results is as good as could be expected. The general form of the variation of (b) is approximately the same as that of (a) , but is apparently slightly more rapid.

TABLE XVI.

Magnetizing Force	Value of (b)	Reference
0.00	360	Table V
2.79	258	Table X
5.95	180	Table XI
10.90	210	Table XII
16.70	234	Table VII
25.90	183	Table VIII
52.90	122	Table IX

To a *first degree of approximation*

$$b = 1.9a,$$

so that the equation

$$\mu = a + 1.9aH$$

or

$$\mu = a \{1 + 1.9H\}$$

and

$$B = aH \{1 + 1.9H\} \text{ to ca. } 0.1\%$$

express the facts as well as could be expected from the extreme smallness of the quantity (b) .

Below, in Table XVII, are given the values of

$$\frac{1.9a}{10} \text{ and } \frac{b}{10}.$$

TABLE XVII.

$\frac{b}{10}$	$\frac{1.9a}{10}$	Diff.
36	36	0
26	26	0
18	22	4
21	21	0
23	21	2.5
18	21	2.5
[12]	[23]	

The equation therefore represents the results [except for the last (b) which deviates greatly] to about 6%. Since there is absolutely no *a priori* reason to expect a connection between the two constants (a) and (b), this result is at least of interest.

The last value of (b) is determined when the magnetization has been pushed very far, and when the downward drift of the readings, mentioned before, is strongest. This may account for the fact that it deviates so far from the above formula.

RELATION TO THE MOLECULAR THEORY.

According to the molecular theory, as developed through the work of Poisson, Weber, Maxwell and Wiedemann, the magnets or molecular groups are to be considered as small permanent magnets. The magnetization of a piece of iron would then correspond to the turning of these magnets into line, and the permeability would therefore depend on the degree of turning, and the number of molecules turned. If then we strongly magnetize a piece of iron, the molecular magnets are permanently set in line and are therefore less free to react for the smaller forces. Hence the permeability for the smaller forces must drop.

The limiting value is to be explained on the idea that the permanent magnetism reaches a limiting value and therefore

the freedom of the molecular magnets can only be lowered a certain definite amount.

On this basis it becomes, however, very difficult to explain the slight tendency to rise when the magnetization becomes very strong. This may be due to some residual effect. In the mind of the author it seems most probable that this slight apparent rise is closely associated with the tendency of the individual readings to decrease when the specimen is strongly magnetized.

III. THE RELATION OF DEMAGNETIZATION IN STEPS TO THE PERMEABILITY FOR SMALL FORCES.

Having studied thus carefully the effect on the permeability of magnetizing the iron after complete demagnetization, it becomes of great interest to study the effect of slow demagnetization in steps after the ring has been strongly magnetized in one direction. That is, instead of starting from zero, we now wish to start from a *negative* point, and come gradually to zero, and then further to *positive* values. We might expect that the second part of the curve would be very similar to the curve found in the last section, but we shall see that, strange to say, it is not. There is a fundamental difference which is very interesting.

METHOD AND APPARATUS.

In the last section it was shown that the form of the permeability function did not change much when the ring was magnetized, and it is therefore unnecessary in the present work to determine a complete curve. It is sufficient to measure the permeability using *one small force*, and then to change the magnetization, and to measure the permeability again.

The choice of a proper size for the measuring force becomes important. If this force is too small, the galvanometer throw will not be large enough to observe accurately, and if the force is too large, it will disturb the permanent magnetism of the iron.

A force of medium size is therefore best, and hence $H = 0.04$ approximately was chosen. This proved entirely satisfactory.

In order to carry out the work conveniently, it was necessary to modify somewhat the arrangement of the apparatus.

Figure 2 shews the complete system. The Ampèremeter (N_1) measures the small force primary current, as formerly, while the Ampèremeter (N_2) serves to measure the large current used to permanently magnetize the iron.

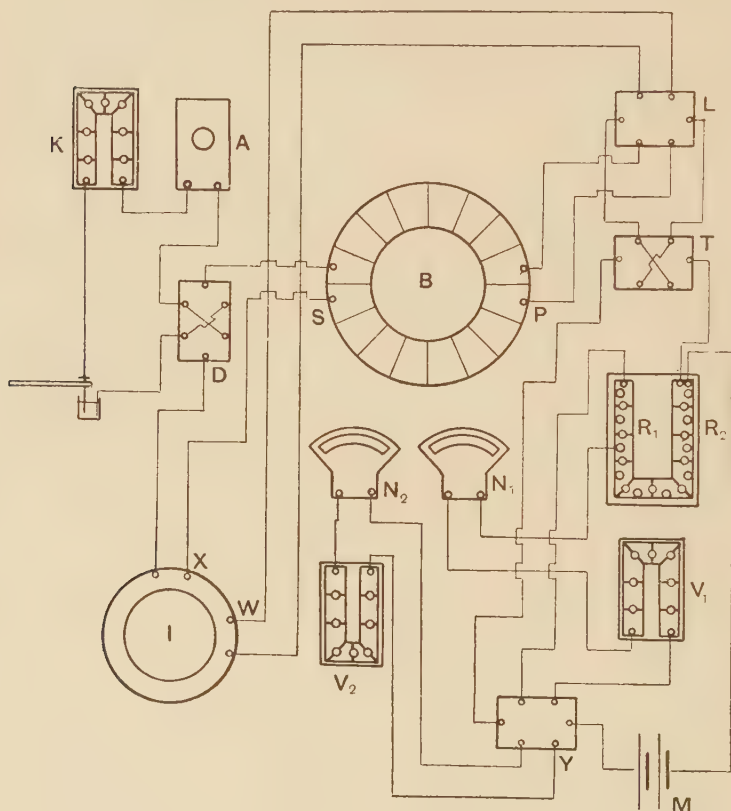


Fig. 2.

The ring was first strongly magnetized by allowing a large measured current to flow for one minute.

The Observations were then made in the following way. The switch (Y) was turned so that the large current instrument

was in series with the primary of the ring, and a demagnetizing current of the desired number of ampères was allowed to flow for exactly one minute. The proper resistance for such a current could be found previously without disturbing the ring, by putting an outside piece of wire in circuit instead of the ring primary. After the desired current had flowed for one minute the circuit was broken. The switch was then thrown for the small current circuit, and *after a number of reversals* the permeability was measured as previously described.

After this had been done, the *large* current circuit was once more switched in, and a slightly *larger* magnetizing current was allowed to flow for one minute.

The same slight tendency of the individual observations to fall was noted here as formerly even after many reversals had previously been made. As mentioned earlier, they *seemed* in general to approach a constant value, and this was usually taken as the final reading.

RESULTS.

In the first series only four points were determined. The results are given below in Table XVIII. The calculation is precisely as previously described [page 12].

A magnetizing current of 3.01 ampères, corresponding to a force of 26.3, was allowed to flow for one minute.

The *demagnetizing* forces were then used in the order named and the permeability measured after the application of each.

TABLE XVIII.

Demag. current	H_0	H	D	B	μ
0.000	0.00	0.04594	85.9	5.070	110.3
0.584	5.10	0.04594	93.1	5.494	119.6
0.972	8.48	0.04594	87.4	5.158	112.3
1.432	12.49	0.04594	86.4	5.099	110.0

The first column represents the demagnetizing current, and (H_0) is the force corresponding. (H) is the small measuring

force used, and (B) the flux density corresponding. (D) is the galvanometer deflection.

It is evident from a glance at the values that the permeability rises first, and then falls, indicating some kind of maximum. It is to be observed that when the value of H_0 is large, the permeability is once more what it was at the beginning, namely 110.0 approximately.

As will be seen this is equal to the permeability for saturation which was obtained in the previous section.

In order to trace out the variation carefully, the ring was once more magnetized with a current of 3 ampères, and the results in Table XIX obtained.

TABLE XIX.

$$H = 0.04573.$$

Demag. current	H_0	D	B	μ
0.0376	0.328	92.6	5.473	119.7
0.0553	0.482	97.2	5.745	125.6
0.0716	0.624	101.9	6.020	131.6
0.0868	0.757	108.9	6.437	140.8
0.1044	0.917	120.8	7.139	156.1
0.1268	1.105	131.8	7.789	170.3
0.1473	1.285	136.2	8.048	176.0
0.1740	1.517	133.9	7.914	173.1

The columns have the same meanings as in the last table.

This table shews well the manner in which the permeability tends toward a maximum. It rises slowly from the saturation value, then more rapidly, passes a point of inflection, then rises more slowly again. It passes a smooth maximum, and begins to drop on the other side.

POSITION OF THE MAXIMUM.

When the maximum of this *demagnetization* curve is compared with the maximum of the *magnetization* curve which was obtained formerly, a remarkable difference is

apparent. *The values of the permeability of the two maxima are not the same.*

The value obtained previously is found by putting small $(H) = 0.04573$ in the first equation for (μ) on page 20. Hence :

Maximum of magnetization curve

$$= 190.4.$$

Whereas the last curve gives

Maximum of demagnetization curve

$$= 176.0.$$

In other words, when the ring passes through the demagnetized state in going from negative to positive values, the

TABLE XX.

Demagnetization Series.

Small $H = 0.04639$.

Correct gal. diff.	Demag. current $C \times 10^{-3}$	H_0	μ	B
87.4	0.00	0.0000	111.4	5.16
90.5	20.65	0.1801	115.4	5.34
95.2	40.10	0.3498	121.4	5.62
100.6	60.90	0.5313	128.2	5.94
107.2	79.80	0.6960	136.8	6.33
119.1	100.10	0.8735	151.9	7.03
126.3	110.50	0.9640	161.1	7.46
132.2	121.10	1.0570	168.6	7.81
134.9	128.60	1.1210	172.0	7.97
135.9	134.00	1.1680	173.3	8.03
136.7	139.40	1.2170	174.4	8.08
137.0	145.50	1.2690	174.7	8.09
137.0	150.30	1.3110	174.7	8.09
136.5	165.00	1.4390	174.1	8.06
134.8	174.00	1.5180	172.0	7.96
131.1	193.00	1.6820	167.2	7.75
120.7	249.00	2.1710	153.9	7.13
106.9	359.00	3.1300	136.3	6.31
94.1	591.00	5.1500	120.0	5.56
90.4	784.00	6.8400	115.5	5.34
87.7	1061.00	9.2600	111.8	5.18
87.1	1856.00	16.1800	111.0	5.14
86.8	4905.00	42.7600	110.7	5.12

permeability is not the same as when the ring is demagnetized by the use of alternating currents. It seems evident that in passing thus from negative to positive magnetization the true zero must be passed through, and yet it is certain from the above results that the iron is not in the same state at each of the maxima.

So remarkable did this result appear that it seemed probable at first that some error had entered into the results. The above complete series of observations was therefore made (Table XX).

The ring was magnetized with a current of 6.00 ampères, corresponding to a magnetizing force of 52.6.

It is readily seen that these measurements confirm those of the other series. The two maxima are 176.0 and 174.7 respectively. The points on the curve near the maximum are very near together, and indicate a *definite, smooth* curve at this point. There can therefore be no doubt that the maximum is about 175.

It is evident from the last series that the part of the curve before the maximum is much steeper than the part after the maximum. That is to say, the *demagnetization* is much more easily produced than the magnetization.

NEW OBSERVATIONS ON THE MAGNETIZATION CURVE.

It was thought best to make a completely new series of observations on the magnetization curve with the same instruments and arrangement of apparatus as used in the case of the last series in order to be absolutely sure that there was really this remarkable difference in the maxima of the two curves.

The ring was therefore carefully demagnetized by means of an alternating current, as previously explained. In this case, however, the smallest alternating current used was less than 0.001 ampère. A complete demagnetization is therefore to be expected.

A complete new series of observations on the magnetization

of the ring was then made. The results were as follows (Table XXI).

TABLE XXI.
Magnetization Series.
 $H = 0.04633$.

D	Mag. current	μ	B	H_0
150.6	0.00000	195.9	907.4	0.0000
149.5	0.00494	194.5	901.2	0.0431
149.4	0.00984	194.3	900.3	0.0859
149.3	0.01493	194.2	899.8	0.1303
149.2	0.02580	194.0	898.9	0.2250
149.2	0.03740	194.0	898.8	0.3260
148.6	0.05170	193.2	895.2	0.4510
148.6	0.08380	193.2	895.2	0.7310
140.0	0.14740	182.0	843.7	1.2860
125.6	0.21100	163.3	756.8	1.8400
118.1	0.25400	153.7	712.0	2.2170
112.2	0.30300	145.8	675.7	2.6400
105.8	0.35000	137.6	637.3	3.0500
102.4	0.40200	133.2	616.9	3.5100
96.4	0.51400	125.4	581.0	4.4900
90.4	0.73800	114.9	544.9	6.4400
86.9	1.09900	113.1	523.4	9.5900
85.9	1.93200	111.8	517.4	16.8500

This new series proves beyond doubt that such a difference in the maxima does exist. This last curve is indeed a close approximation to the curve obtained in Section II. by plotting the constants (a).

It should be remembered however that in the present curve the ordinates do not correspond to the constants (a), but to values corresponding to small $H = 0.04633$.

CAUSE OF THE DIFFERENCE IN MAXIMA.

The problem then presents itself, "How can the difference of maxima be explained?" In the opinion of the author it can only be explained by the assumption that there is more than

one magnetizable element existing in the iron. These elements do not react with equal readiness to the demagnetizing force, and hence one element passes through zero and becomes positively magnetized before the other (or others) have reached zero. Thus at *no instant* can the iron be said to be *completely demagnetized*, because, as we may say, the elements are not in phase with each other.

If there were only *one* element to the magnetization, it would be impossible to pass from positive to negative magnetization without passing through zero.

The ring form of the iron would also produce a phenomenon of this kind, because the force nearer the axis must be greater than that further from it. The iron in the inner part of the ring must therefore become demagnetized first, and will become positive before the magnetization of the outer part of the ring has reached zero.

An inspection of the Tables will shew that this is probably *only a part* of the whole effect, for if it were all, the two curves would coincide after the outer iron of the ring had passed through zero magnetization, and this point would be only about 0.4 beyond the maximum.

If the curves are plotted it will be seen that they do not meet in this way, but seem to approach each other at a much greater distance.

It is therefore to be concluded that there is in the iron more than one magnetizable element; these are demagnetized with unequal readiness, and therefore do not all become zero at the same time.

The facts would also be explained by assuming varying degrees of stability to the polarized molecular groups.

These seem the only explanations to the somewhat remarkable experimental result.

OUTLINE OF CONCLUSIONS AND RESULTS FOR THE ENTIRE INVESTIGATION.

The principal conclusions and results of the entire investigation may be outlined as follows:

(i) The permeability for very small forces is never a constant within the region measured. For values of the magnetizing force between 0.002 and 0.01 it is expressed (for this specimen) by the equation

$$\mu = 169.4 + 580H$$

to an accuracy of 0.05 %;

and for (H) between 0.02 and 0.1 by

$$\mu = 167.0 + 816H - 700H^2$$

to an accuracy of 0.3 %.

The quadratic form of (μ) has not previously been determined.

(ii) The effect of permanent magnetization is to *lower* the permeability for small forces, at first rapidly and then slowly, reaching a saturation value when the force used to give the permanent magnetism is about 11.0 absolute c.g.s. units. This represents of course the variation in the constant (a) of the formula

$$\mu = a + bH.$$

The constant (b) in this equation also changes under magnetization, and it has been found that (except for the greatest magnetization)

$$b = 1.9a \text{ approximately.}$$

The equation for μ can therefore be written with one constant

$$\mu = a(1 + 1.9H)$$

with an accuracy of about 0.1 %.

(iii) When the iron is strongly magnetized, and is then demagnetized in steps, the permeability increases rapidly,

passes through a rounded maximum, and then decreases less rapidly down to the limiting value.

The remarkable result is however obtained that the maximum passed through in this way is much lower than the value for the demagnetized ring.

The conclusion is reached that there must exist several elements to the magnetization in the iron, which do not undergo demagnetization equally easily, and which do not therefore pass through zero at the same time. These may be different molecular groups or the same groups with different degrees of stability.

For a fuller statement of this conclusion, see page 39.

I, DANIEL FROST COMSTOCK, was born on Aug. 14th, 1883, in Newport, R.I., U.S.A. I attended the primary schools of Marion, Indiana, and later the High School in that city, where I remained three years. In the year 1900 I entered the Massachusetts Institute of Technology in Boston and studied there continuously for the next four years. At the end of that time, having passed the necessary examinations and having presented a dissertation, I obtained a diploma in Physics. I remained one year at this institution as Assistant and was then promoted to the grade of Instructor, appointed Fellow and given two years' leave of absence. The summer semester of 1905 I spent at the University of Berlin and the next two semesters at the Eidgenössisches Polytechnikum in Zürich where I performed the present investigation under the direction of Prof. Dr H. F. Weber. In July 1906 I took my doctor examinations at the University of Basel.

